Department of Mathematics

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Comprehensive Examination–Option I

2017 Spring

Complex Analysis

1. **Let f be analytic in the open set C, and C a closed rectifiable curve in with z . C. Prove**

$$\int_{C} \frac{f'(z)\,dz}{z \quad z} = \int_{C} \frac{f(z)\,dz}{(z \quad z)^2}$$

2. **Let f be an entire function such that there exists M R such that $|f(z)| \quad |z|^4$ for each $|z| \quad M$. Prove that f is a polynomial in z.**

3. **Find a bijective analytic map f : D**

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Applied Analysis

1. Give the general solution of the following system of equations.

$$\mathbf{x}' = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 4e^{t} \\ 5 \quad + 2 \end{bmatrix}$$

2. There exist infinitely many functions $M(\quad y)$ such that the first order ODE

$$M(\quad y)\, d \quad + (\quad y + 3 \quad {}^{2}y \quad y^{3})\, dy = 0$$

is exact. Find one explicit such $M(\quad y)$.

3. <u>Definition</u>. For a sequence $(\quad _n)$ we say that

$$\lim_{n \to \infty} \quad _n =$$

if for each M 0 there exists N N such that $_n$ M n N.
Use T5TJ/R13(8)23.6121(e)-n

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Numerical Analysis

1. Let $g(\ ) = \frac{1}{3}e^{x}$.

   (a) Prove that there exist exactly two real solutions of the equation $= g(\ )$.

   (b) Let be the lesser of the two solutions. Find an interval [ ] such that fixed-point iteration on $g(\ )$ will produce a sequence $(\ _n)_{n=}^{\infty}$ of iterates which, for any [ ], converges to . Use the fixed-point iteration theorem to justify your answer.

2. Recall that the truncation errors $E_{\ n}$ and $E_{\ m}$ for approximating

$$\int_a^b f$$

by the trapezoidal rule with n equal-sized subintervals of [ ], and by Simpson's rule with equal-sized subintervals of [ ], respectively, are given by

$$E_{\ n} = \frac{(\ \ )^{3}}{12n^2} f''(\ _T) \quad \text{some } _T \ (\ \ ) \text{ and}$$

$$E_{\ m} = \frac{(\ \ )^5}{180\ ^4} f^{(4)}(\ _S) \quad \text{some } _S \ (\ \ )$$

Show that Simpson's rule with $= 18$ yields a more accurate approximation of

$$\int^1 e^{x^2}\, d$$

than the trapezoidal rule with $n = 180$. That is, show that for $[\ \ ] = [0\ 1]$ and $f(\ ) = e^{x^2}$ we have $|E_{\ 1}|\quad |E_{\ 1}|$.

$e \quad n A \quad fD$

$n\ f\ d \quad f\ dLLe$

3. Let A be a nonsingular $n \quad n$ real matrix with

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Linear Programming

1. **Consider the following problem.**

$$
\begin{array}{rl}
\textbf{minimize} & 5\ {}_1 + \quad {}_2 + 4\ {}_3 \\
\textbf{subject to} & 3\ {}_1 + 2\ {}_2 \quad 3\ {}_3
\end{array}
$$

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Probability

1. **Ulysses and Iolanthe are going to play checkers. They enjoy playing "matches" wherein the first to win two games wins the match. In each game either Ulysses wins or Iolanthe wins; that is, there are no "draws". It turns out that they are evenly matched players, but there is an advantage to moving first. So the player who goes first wins 52% of the games. They flip a coin to determine who goes first in the first game, and Ulysses wins. He will move first in the first and third games (if the third game is needed for the match), while Iolanthe will go first in the second game. For this problem we assume that the games are independent**
   a. **Using the notation UU to indicate that Ulysses won both the first and second games; and UII to indicate that Ulysses won the first game, but Iolanthe won the second and third games, write the sample space for the match.**
   b. **Calculate the probabilities for all the events in the sample space, and give a table indicating the probability mass function for the match.**
   c. **Let X represent the number of games Ulysses wins in the match. Using your answer to part b, give the probability mass function for X.**
   d. **Let Y represent the number of games Iolanthe wins in the match. Using your answer to part b, give the probability mass function for Y.**
   e. **Let Z represent the number of games played in the match. Using your answer to part b, give the probability mass function for Z.**
   f. **Let A represent the winner of the match. Using your answers to b, c, d, or e, as appropriate, give the probability mass function for A.**
   g. **Had Iolanthe won the coin toss and gone first in the first and third games, what is the probability she would have won the match?**

2. **Let X and Y be jointly distributed random variables with the following joint probability density function:**

$$f_{X,Y}(\quad y) = \begin{cases} +y \quad 0 \qquad 1 \; 0 \quad y \quad 1 \\ 0 \qquad\qquad \text{otherwise} \end{cases}$$

   a. **Find the marginal density function for Y $f_Y(y)$.**
   b. **Find $E(Y)$.**
   c. **Find the conditional density function of X for a given y $f_{X|Y}(\; |y)$.**
   d. **Find $E(X|Y = y)$.**
   e. **Are X and Y independent? Justify your answer completely. If X and Y are not independent, then find $Co \; (X \; Y)$.**

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Probability — continued

3. Suppose we have a class of sixteen students, and each student turns in one homework for the class. The homework is graded and needs to be returned to the students. Assume that the homeworks do not have student identification on them. Let the random variable X be the number of students who receive their own homework after shu□ing.
   (a) Find the expected number of students who receive their own homework back; that is, find $E(X)$.
   (b) Find the variance of X $V(X)$.
   (c) Extend part (a) for a class of n students and generalize the results. That is, find $E(X)$, the expected number of students who correctly receive their homework back for a class with n students.

4. Consider a football game between two rival teams: Team 0 and Team 1. Suppose Team 0 wins 65% of the time and Team 1 wins the remaining matches. Among the games won by Team 0 only 30% of them come from playing on Team 1's football field. On the other hand, 75% of the victories for Team 1 are obtained while playing at home.
   (a) Let $Y = 0$ or 1 be a random variable that represents the winner of the match. Let $X = 0$ or 1 be the random variable that represents the team hosting the match. Using appropriate probability notation define the probabilities given above,
   (b) If Team 1 is to host the next match between the two teams, what is the conditional probability that Team 1 wins the next match it will be hosting? Compute $P(Y = 1|X = 1)$.
   (c) If Team 1 hosts the next match between the two teams, which team will most likely emerge as the winner?
   (d) If Team 0 is to host the next match between the two teams, what is the conditional probability that Team 0 wins the next match it will be hosting? Compute $P(Y = 0|X = 0)$.
   (e) If Team 0 hosts the next match between the two teams, which team will most likely emerge as the winner?
   (f) Which team is the better competitor while playing at home?